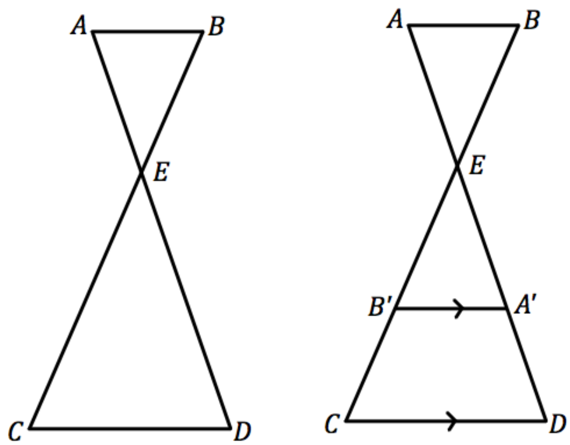
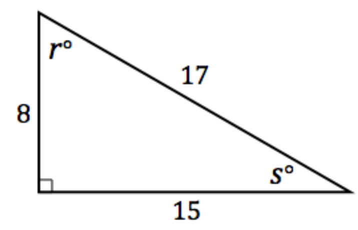
Name ____ Date ______________________

1. In the figure below, rotate $\triangle EAB$ about E by $180°$ to get $\triangle EA'B'$. If $\overline{A'B'} \parallel \overline{CD}$, prove that $\triangle EAB \sim \triangle EDC$.



2. Answer the following questions based on the diagram below.



a. Find the sine and cosine values of angles r and s. Leave answers as fractions.

$\sin r° =$ $\sin s° =$

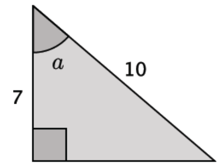
$\cos r° =$ $\cos s° =$

$\tan r° =$ $\tan s° =$

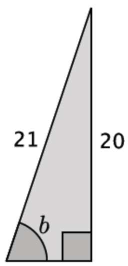
b. Why is the sine of an acute angle the same value as the cosine of its complement?

c. Determine the measures of the angles to the nearest tenth of a degree, in the right triangles below.

i. Determine the measure of $\angle a$.



ii. Determine the measure of $\angle b$.



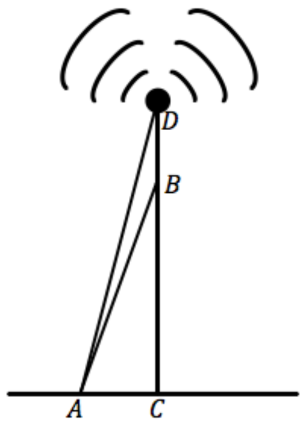
iii. Explain how you were able to determine the measure of the unknown angle in part (i) or part (ii).

d. A ball is dropped from the top of a 45 ft building. Once the ball is released a strong gust of wind blew the ball off course and it dropped 4 ft from the base of the building.

i. Sketch a diagram of the situation.

ii. By approximately how many degrees was the ball blown off course? Round your answer to the nearest whole degree.

3. A radio tower is anchored by long cables called guy wires, such as AB in the figure below. Point A is 250 m from the base of the tower, and $\angle BAC = 59°$.



a. How long is the guy wire? Round to the nearest tenth.

b. How far above the ground is it fastened to the tower?

c. How tall is the tower, DC, if $\angle DAC = 71°$?

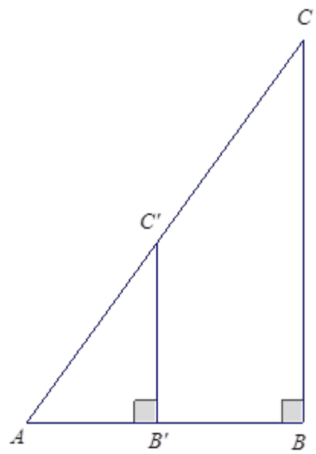
4. The following problem is modeled after a surveying question developed by a Chinese mathematician during the Tang Dynasty in the seventh century A.D.

   A building sits on the edge of a river. A man views the building from the opposite side of the river. He measures the angle of elevation with a hand-held tool and finds the angle measure to be $45°$. He moves 50 feet away from the river and re-measures the angle of elevation to be $30°$.

   What is the height of the building? From his original location, how far away is the viewer from the top of the building? Round to the nearest whole foot.

5. Prove the Pythagorean theorem using similar triangles. Provide a well-labeled diagram to support your justification.

6. In right triangle $\triangle ABC$ with $\angle B$ a right angle, a line segment $B'C'$ connects side AB with the hypotenuse so that $\angle AB'C'$ is a right angle as shown. Use facts about similar triangles to show why $\cos C' = \cos C$.



7. Terry said, "I will define the zine of an angle x as follows. Build an isosceles triangle in which the sides of equal length meet at angle x. The zine of x will be the ratio of the length of the base of that triangle to the length of one of the equal sides." Molly said, "Won't the zine of x depend on how you build the isosceles triangle?"

   a. What can Terry say to convince Molly that she need not worry about this? Explain your answer.

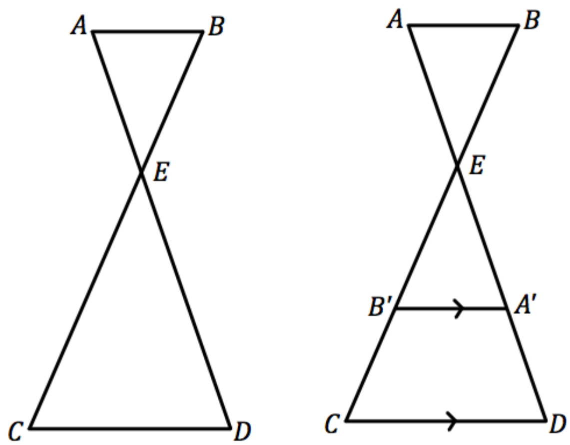
   b. Describe a relationship between zine and sin.

A Progression Toward Mastery					
Assessment Task Item		**STEP 1** **Missing or incorrect answer and little evidence of reasoning or application of mathematics to solve the problem.**	**STEP 2** **Missing or incorrect answer but evidence of some reasoning or application of mathematics to solve the problem.**	**STEP 3** **A correct answer with some evidence of reasoning or application of mathematics to solve the problem, OR an incorrect answer with substantial evidence of solid reasoning or application of mathematics to solve the problem.**	**STEP 4** **A correct answer supported by substantial evidence of solid reasoning or application of mathematics to solve the problem.**
1	**G-SRT.B.5**	Student shows little or no understanding of how to prove $\triangle EAB \sim \triangle EDC$.	Student makes an error in the rotation or it is not shown and makes an error in the justification.	Student makes an error in the rotation or it is not shown, but the rest of the justification is correct. OR Student correctly makes the rotation but makes an error in the justification.	Student correctly explains and proves $\triangle EAB \sim \triangle EDC$.
2	**a–b** **G-SRT.C.7**	Student makes one or more errors in part (a) and does not answer or provides poor reasoning for part (b).	Student makes one error in part (a) and makes an error in his or her reasoning for part (b).	Student correctly answers part (a) but makes an error in his or her reasoning for part (b).	Student correctly answers parts (a) and (b).
	c **G-SRT.C.7**	Student makes little or no attempt to find the angle measures.	Student correctly identifies the measure of each angle in part (i) and part (ii) but does not have an explanation in part (iii) OR Student correctly identifies the measure of one of the angles in part (i) or part (ii) and has a weak explanation for part (iii).	Student correctly identifies the measure of each angle in part (i) and part (ii) but has a weak explanation in part (iii) or student correctly identifies the measure of one of the angles in part (i) or part (ii) and has a complete explanation for part (iii).	Student correctly identifies the measure of each angle in part (i) and part (ii) and has a complete explanation for how the answer was determined.

	d **G-SRT.C.7**	Student makes little or no attempt to complete the problem. A sketch may be drawn, but not accurate for the situation.	Student draws a sketch and makes a major error in the calculation of the angle. For example, the student may have used a function other than arctan.	Student draws an accurate sketch but may have made a mathematical error leading to an incorrect answer.	Student draws an accurate sketch and has identified the correct angle measure.
3	**a–c** **G-SRT.C.8**	Student makes at least one computational or conceptual error in all three parts. OR Student makes at least one computational and one conceptual error in any two parts.	Student makes one computational or one conceptual error in any two parts. OR Student makes more than one computational or conceptual error in any one of the three parts.	Student makes one computational or conceptual error in any one of the three parts.	Student correctly answers all three parts.
4	**G-SRT.C.8**	Student makes three or more computational or conceptual errors.	Student makes two computational or conceptual errors.	Student makes one computational or conceptual error.	Student correctly finds the height of the building and the distance of the viewer to the top of the building.
5	**G-SRT.B.4**	Student shows little or no understanding of how similar triangles can be used to prove the Pythagorean theorem.	Student provides a correctly labeled diagram but offers an inaccurate justification towards proving the Pythagorean theorem.	Student provides a correctly labeled diagram and shows how each of the sub-triangles have lengths in proportion to the corresponding lengths of the large triangle but does not tie these ideas together to show why the Pythagorean theorem must be true.	Student clearly demonstrates how to use similar triangles to prove the Pythagorean theorem and includes an accurately drawn and labeled diagram.
6	**G-SRT.C.6**	Student shows little or no understanding of similar triangles; student is unable to coherently show why $\triangle ABC \sim \triangle AB'C'$.	Student cites that $\triangle ABC \sim \triangle AB'C'$ but does not explicitly state facts about similar triangles to further explain why $\cos C' = \cos C$.	Student uses facts about similar triangles to show why $\frac{B'C'}{AC'} = \frac{BC}{AC}$ but does not explicitly conclude that $\cos C' = \cos C$.	Student correctly demonstrates that $\cos C' = \cos C$.
7	**a–b** **G-SRT.C.6**	Student shows little or no understanding of similar triangles.	Student shows evidence of understanding but lacks clarity in the reasoning for both parts (a) and (b).	Student shows evidence of understanding but lacks clarity in the reasoning for either part (a) or part (b).	Student provides accurate and well-reasoned responses for both parts (a) and (b).

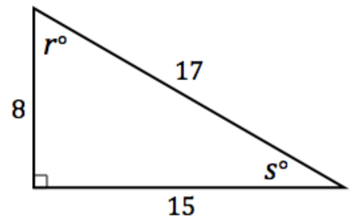
Name ______________________________ Date ______________

1. In the figure below, rotate $\triangle EAB$ about E by $180°$ to get $\triangle EA'B'$. If $\overline{A'B'} \parallel \overline{CD}$, prove that $\triangle EAB \sim \triangle EDC$.



Triangles EAB and EA'B' are congruent since a 180° rotation is a rigid motion. Since $\overline{A'B'} \parallel \overline{CD}$, $m\angle EB'A' = m\angle ECD$, and $m\angle B'EA' = m\angle CED$. So $\triangle EA'B' \sim \triangle EDC$ by AA similarity criteria, and $\triangle EAB \sim \triangle EDC$ by the transitive property of similarity.

2. Answer the following questions based on the diagram below.



a. Find the sine and cosine values of angles r and s. Leave answers as fractions.

$\sin r^\circ = \frac{15}{17}$ $\sin s^\circ = \frac{8}{17}$

$\cos r^\circ = \frac{8}{17}$ $\cos s^\circ = \frac{15}{17}$

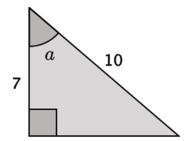
$\tan r^\circ = \frac{15}{8}$ $\tan s^\circ = \frac{8}{15}$

b. Why is the sine of an acute angle the same value as the cosine of its complement?

By definition sine is the ratio of the opposite side: hypotenuse, and cosine is the ratio of the adjacent side: hypotenuse; since the opposite side of an angle is the adjacent side of its complement, $\sin\theta = \cos(90-\theta)$.

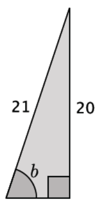
c. Determine the measures of the angles to the nearest tenth of a degree, in the right triangles below.

i. Determine the measure of $\angle a$.



$m\angle a \approx 45.6^\circ$

ii. Determine the measure of $\angle b$.



$m\angle b \approx 72.2^\circ$

iii. Explain how you were able to determine the measure of the unknown angle in part (i) or part (ii).

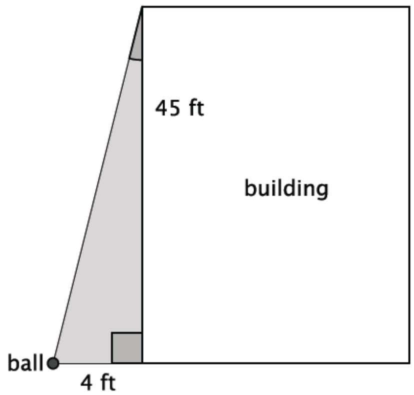
For part (i), students should state that they had to use arccos to determine the unknown angle because the information given about the side lengths included the side adjacent to the unknown angle and the hypotenuse.

For part (ii), students should state that they had to use arcsin to determine the unknown angle because the information given about the side lengths included the side opposite to the unknown angle and the hypotenuse.

d. A ball is dropped from the top of a 45 ft building. Once the ball is released a strong gust of wind blew the ball off course and it dropped 4 ft from the base of the building.

i. Sketch a diagram of the situation.

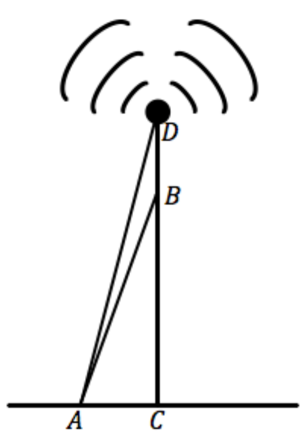
Sample sketch shown below.



ii. By approximately how many degrees was the ball blown off course? Round your answer to the nearest whole degree.

The wind blew the ball about 5° off course.

3. A radio tower is anchored by long cables called guy wires, such as $\overline{AB}$ in the figure below. Point A is 250 m from the base of the tower, and $\angle BAC = 59°$.



a. How long is the guy wire? Round to the nearest tenth.

$\cos 59 = \frac{250}{AB}$

$AB = \frac{250}{\cos 59}$

$AB \approx 485.4$ m

b. How far above the ground is it fastened to the tower?

$\text{Tan } 59 = \frac{BC}{250}$

$BC = 250 \tan 59$

$BC \approx 416.1$ m

c. How tall is the tower, $\overline{DC}$, if $\angle DAC = 71^\circ$?

$\tan 71 = \frac{DC}{250}$

$DC = 250 \tan 71$

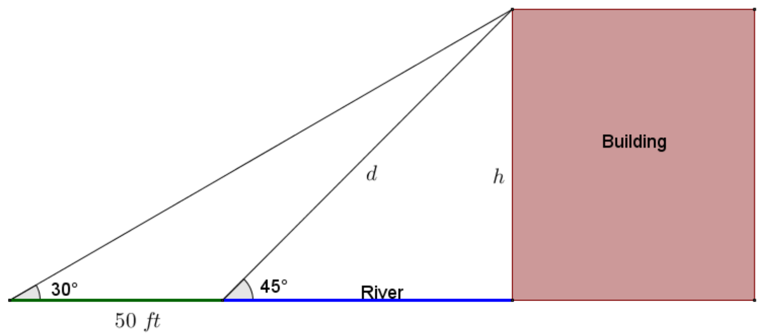
$DC \approx 726.1$ m

4. The following problem is modeled after a surveying question developed by a Chinese mathematician during the Tang Dynasty in the seventh century A.D.

A building sits on the edge of a river. A man views the building from the opposite side of the river. He measures the angle of elevation with a hand-held tool and finds the angle measure to be $45°$. He moves 50 feet away from the river and re-measures the angle of elevation to be $30°$.

What is the height of the building? From his original location, how far away is the viewer from the top of the building? Round to the nearest whole foot.

The angle of depression from the top of the building to the man's original spot is also 45°, and the angle of depression to his final position is 60°, so the difference of the angles is 15°. Let d represent the distance from the man's position at the edge of the river to the top of the building, and let h represent the height of the building in feet.



Using the law of sines:

$$\frac{\sin 30}{d} = \frac{\sin 15}{50}$$

$$d = \frac{50 \sin 30}{\sin 15} \approx 96.6$$

The distance d is approximately 96.6 feet.

By the Pythagorean theorem, the distance $d = h\sqrt{2}$.

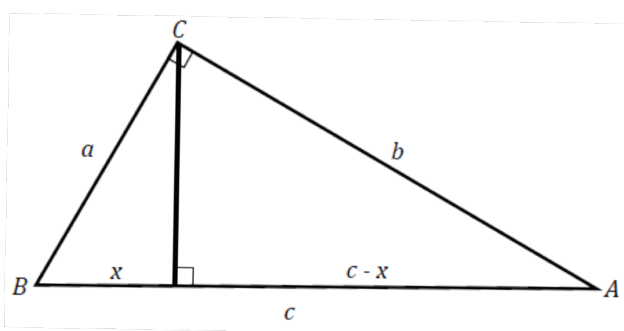
$$50\frac{\sin 30}{\sin 15} = h\sqrt{2}$$

$$\frac{25}{\sin 15\left(\sqrt{2}\right)} = h$$

$$h \approx 68.3$$

The distance from the viewer to the top of the building is approximately 97 ft., and the height of the building is approximately 68 ft.

5. Prove the Pythagorean theorem using similar triangles. Provide a well-labeled diagram to support your justification.



A right triangle $\triangle ABC$ has side lengths a, b, and c. An altitude is drawn from C to the opposite side, dividing c into lengths x and c–x. Since the altitude from C divides the triangle into to smaller similar right triangles by the AA criterion, then:

$$\frac{a}{x} = \frac{c}{a}$$

$$a^2 = cx$$

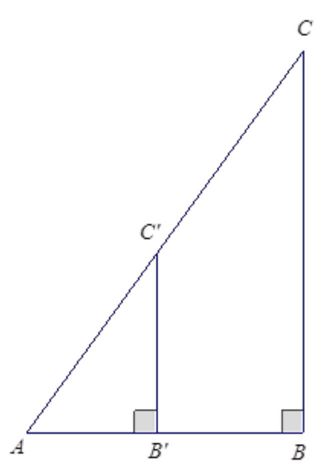
$$\frac{b}{c - x} = \frac{c}{b}$$

$$b^2 = c(c - x)$$

$$a^2 + b^2 = cx + c(c - x) = c^2$$

$$a^2 + b^2 = c^2$$

6. In right triangle $\triangle\, ABC$ with $\angle B$ a right angle, a line segment $B'C'$ connects side AB with the hypotenuse so that $\angle AB'C'$ is a right angle as shown. Use facts about similar triangles to show why $\cos C' = \cos C$.



By the AA criterion, $\triangle ABC \sim \triangle AB'C'$. Let r be the scale factor of the similarity transformation. Then $B'C' = r \cdot BC$ and $AC' = r \cdot AC$. Thus,

$$\cos C' = \frac{B'C'}{AC'} = \frac{r \cdot BC}{r \cdot AC} = \frac{BC}{AC} = \cos C$$

$$\cos C' = \cos C$$

7. Terry said, "I will define the zine of an angle x as follows. Build an isosceles triangle in which the sides of equal length meet at angle x. The zine of x will be the ratio of the length of the base of that triangle to the length of one of the equal sides." Molly said, "Won't the zine of x depend on how you build the isosceles triangle?"

a. What can Terry say to convince Molly that she need not worry about this? Explain your answer.

Isosceles triangles with vertex angle x are similar to each other; therefore, the value of zine x, the ratio of the length of the base of that triangle to the length of one of the equal sides, is the same for all such triangles.

b. Describe a relationship between zine and sin.

$\text{zine } x = 2 \sin \frac{1}{2}x$