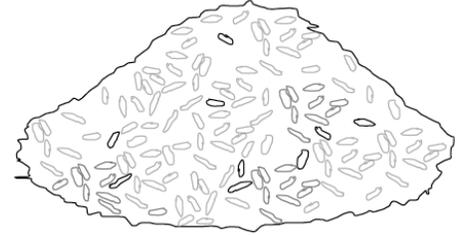


- c. Give an explanation of how to use the volume formula of a pyramid to show that the volume formula of a circular cone is $\frac{1}{3}\pi r^2 h$, where r is the radius of the cone and h is the height of the cone.
2. A circular cylinder has a radius between 5.50 and 6.00 centimeters and a volume of 225 cubic centimeters. Write an inequality that represents the range of possible heights the cylinder can have to meet this criterion to the nearest hundredth of a centimeter.

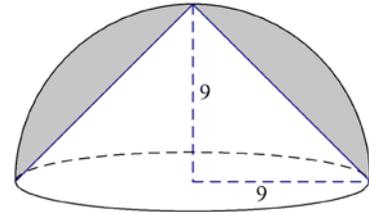
3. A machine part is manufactured from a block of iron with circular cylindrical slots. The block of iron has a width of 14 in., a height of 16 in., and a length of 20 in. The number of cylinders drilled out of the block is determined by the weight of the leftover block, which must be less than 1,000 lb.
- a. If iron has a weight of roughly 491 lb/ft^3 , how many cylinders with the same height as the block and with radius 2 in. must be drilled out of the block in order for the remaining solid to weigh less than 1,000 lb.?
- b. If iron ore costs \$115 per ton (1 ton = 2200 lb.) and the price of each part is based solely on its weight of iron, how many parts can be purchased with \$1,500? Explain your answer.

4. Rice falling from an open bag piles up into a figure conical in shape with an approximate radius of 5 cm.
- a. If the angle formed by the slant of the pile with the base is roughly 30° , write an expression that represents the volume of rice in the pile.



- b. If there are approximately 20 grains of rice in a cubic centimeter, approximately how many grains of rice are in a 4.5-kilogram bag of rice?

5. In a solid hemisphere, a cone is removed as shown. Calculate the volume of the resulting solid. In addition to your solution, provide an explanation of the strategy you used in your solution.



6. Describe the shape of the cross-section of each of the following objects.

Right circular cone:

- a. Cut by a plane through the vertex and perpendicular to the base

Square pyramid:

- b. Cut by a plane through the vertex and perpendicular to the base
- c. Cut by a vertical plane that is parallel to an edge of the base but not passing through the vertex

Sphere with radius r :

- d. Describe the radius of the circular cross-section created by a plane through the center of the sphere.

- e. Describe the radius of the circular cross-section cut by a plane that does not pass through the center of the sphere.

Triangular Prism:

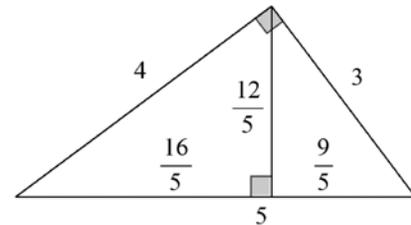
- f. Cut by a plane parallel to a base

 - g. Cut by a plane parallel to a face
- 7.
- a. A 3×5 rectangle is revolved about one of its sides of length 5 to create a solid of revolution. Find the volume of the solid.

b. A 3-4-5 right triangle is revolved about a leg of length 4 to create a solid of revolution. Describe the solid.

c. A 3-4-5 right triangle is revolved about its legs to create two solids. Find each of the volumes of the two solids created.

d. Show that the volume of the solid created by revolving a 3-4-5 triangle about its hypotenuse is $\frac{48}{5}\pi$.



A Progression Toward Mastery

| Assessment Task Item | STEP 1 Missing or incorrect answer and little evidence of reasoning or application of mathematics to solve the problem. | STEP 2 Missing or incorrect answer but evidence of some reasoning or application of mathematics to solve the problem. | STEP 3 A correct answer with some evidence of reasoning or application of mathematics to solve the problem, <u>OR</u> an incorrect answer with substantial evidence of solid reasoning or application of mathematics to solve the problem. | STEP 4 A correct answer supported by substantial evidence of solid reasoning or application of mathematics to solve the problem. | |
|----------------------|--|--|--|---|--|
| 1 | a G-GMD.A.1 | Student incorrectly states the volume formula. Response is not coherent or does not provide reasoning as to why the formula works or response not attempted. | Student correctly states the volume formula but does not explain why it works. <u>OR</u> Student incorrectly states the volume formula, but provides reasoning as to why the formula works. | Student correctly states the volume formula and provides some evidence of understanding why the formula works. | Student correctly states the volume formula and provides substantial reasoning as to why the formula works. |
| | b G-GMD.A.1 | Student demonstrates little or no understanding of the one-third factor in the volume formula. Student may have left item blank. | Student demonstrates some understanding of the one-third factor in the volume formula. | Student demonstrates understanding of the one-third factor in the volume formula. | Student demonstrates clear and coherent reasoning of understanding of the one-third factor in the volume formula. More than one explanation may have been given. |
| | c G-GMD.A.1 | Student demonstrates little or no understanding of how the volume of a pyramid can be used to establish the volume formula for a circular cone. | Student demonstrates an understanding of how the volume of a pyramid can approximate the volume of a circular cone, but the response lacks specificity regarding how the polygonal base approximates the circular base and fails to mention the volume formula of a pyramid. | Student demonstrates an understanding of how the volume of a pyramid can approximate the volume of a circular cone, including the formula of the volume of a pyramid, but the response lacks specificity regarding how the polygonal base approximates the circular base. | Student demonstrates clear and coherent reasoning of understanding of how the volume of a pyramid can approximate the volume of a circular cone. |

| | | | | | | |
|---|-----------|---|---|--|---|--|
| 2 | G-GMD.A.3 | Student does not provide a coherent response. <u>OR</u> Student does not attempt to provide a response. | Student provides a response that contains two computational errors or two conceptual errors, or one computational and one conceptual error. | Student provides a response that contains one computational error or one conceptual error, such as failing to find one extreme of the range of heights. | Student correctly identifies range of possible heights rounded to the nearest hundredth of a centimeter in an inequality. | |
| | | 3 | a | G-MG.A.2 | Student provides an incorrect answer and does not show work. | Student provides a response that contains two computational errors or two conceptual errors, or one computational and one conceptual error. |
| | b | G-MG.A.3 | Student provides an incorrect answer and does not show work. | Student provides a response that contains two computational errors or two conceptual errors, or one computational and one conceptual error. | Student provides a response that contains one computational error, such as an incorrect cost per pound of iron, or one conceptual error, such as incorrect conversion of units. | Student correctly determines the number of parts that can be purchased with \$1,500 and shows a complete solution. |
| 4 | a | G-MG.A.1 | Student demonstrates little or no understanding of how the volume of a pyramid can be used to establish the volume formula for a circular cone. | Student demonstrates an understanding of how the volume of a pyramid can approximate the volume of a circular cone, but the response lacks specificity regarding how the polygonal base approximates the circular base and fails to mention the volume formula of a pyramid. | Student demonstrates an understanding of how the volume of a pyramid can approximate the volume of a circular cone, including the formula of the volume of a pyramid, but the response lacks specificity regarding how the polygonal base approximates the circular base. | Student demonstrates clear and coherent reasoning of understanding of how the volume of a pyramid can approximate the volume of a circular cone. |
| | b | G-MG.A.2 | Student demonstrates little or no understanding of how to determine the number of grains of rice in the pile. | Student provides a response that contains one computational and one conceptual error, such as incorrect conversion of units. | Student provides a response that contains one computational error or one conceptual error, such as incorrect conversion of units. | Student correctly determines the number of grains of rice in the pile and shows a complete solution. |

| | | | | | |
|---|--------------------------------|---|---|---|---|
| 5 | G-GMD.A.3 | Student makes a computational error leading to an incorrect answer; additionally, the reasoning is not correct. | Student provides a response that contains a computational error leading to an incorrect answer but uses correct reasoning. | Student provides a correct solution but does not include an explanation as to how they arrived at their answer. | Student provides a correct solution which includes a clear explanation of how the problem is approached and solved. |
| 6 | a–g G-GMD.B.4 | Student provides a correct response to one or less parts. | Student provides a correct response to three parts. | Student provides a correct response to five parts. | Student provides a correct response to all seven parts. |
| 7 | a G-GMD.B.4 | Student shows little or no attempt to answer the question. | Student provides a response that contains one conceptual error, such as calculating the volume of a rectangular prism. | Student provides a response that contains one computational error, such as using an incorrect height or radius leading to an answer of 75π . | Student provides a correct solution and final answer of 45π . |
| | b G-GMD.B.4 | Student identifies the shape as a triangle or other two-dimensional figure or left the item blank. | Student misidentifies the shape formed but correctly states it as a three-dimensional solid. | Student concludes that the shape formed by the revolution is a right circular cone or cone. | Student provides a complete sentence identifying the solid formed as a right circular cone or cone. |
| | c G-GMD.B.4 | Student provides incorrect answers for both solids, and it is not evident that formulas were used. | Student used the correct formulas for volume but makes computational errors leading to incorrect volumes for both solids. | Student correctly calculates the volume of one of the solids but may have made a computational error leading to an incorrect volume for the second solid. | Student correctly provides the volume of both solids (i.e., 12π and 16π). |
| | d G-GMD.B.4 | Student made little or no attempt to answer the question. | Student provides a solution with the correct steps but makes one conceptual error leading to an incorrect answer, such as using an incorrect formula. | Student provides a solution with the correct steps but makes one computational error leading to an incorrect answer. | Student correctly identifies the volume as $\frac{48}{5}\pi$ and includes a complete solution. |

Name _____

Date _____

1.

- a. State the volume formula for a cylinder. Explain why the volume formula works.

$V = Bh$, where B is the area of the base.
Since a cylinder is comprised of congruent disks (circles), then the height of the cylinder is produced by the "stack" of disks. Congruent figures have equal area, so multiplying $B \times h$ gives the volume of a cylinder.

- b. The volume formula for a pyramid is $\frac{1}{3}Bh$, where B is the area of the base and h is the height of the solid. Explain where the $\frac{1}{3}$ comes from in the formula.

If you consider a unit cube and a pyramid whose base is equal to 1×1 and whose height is $\frac{1}{2}$, it would take 6 such pyramids to equal the volume of the cube. If you consider just half the volume of the cube, then we have a solid with volume $\frac{1}{6}$, the pyramid, when compared $\frac{1}{6} = ? \times \frac{1}{2}$, the missing factor is $\frac{1}{3}$. So the volume of the pyramid is $\frac{1}{3}$ of the volume of the rectangular prism that is the same base and same height.

- c. Give an explanation of how to use the volume formula of a pyramid to show that the volume formula of a circular cone is $\frac{1}{3}\pi r^2 h$, where r is the radius of the cone and h is the height of the cone.

We can approximate the cone with a pyramid, so that the base of the pyramid is an n -sided polygon that approximates the base of the circular cone, and the height is the same as that of the cone. The volume of the pyramid can be expressed as $\frac{1}{3}B_n h$ where B_n is the area of the polygonal base, and h is the height. As the number of sides approaches infinity, the value of B_n approaches πr^2 , where r is the radius of the circle within which the polygonal base is inscribed.

2. A circular cylinder has a radius between 5.50 and 6.00 centimeters and a volume of 225 cubic centimeters. Write an inequality that represents the range of possible heights the cylinder can have to meet this criterion to the nearest hundredth of a centimeter.

$$5.50 < r < 6.00, \quad V = 225 \text{ cm}^3$$

$$V = \pi r^2 h$$

$$225 = \pi (5.5)^2 h$$

$$2.37 \approx h$$

$$V = \pi r^2 h$$

$$225 = \pi (6.00)^2 h$$

$$1.99 \approx h$$

$$2.37 < h < 1.99$$

3. A machine part is manufactured from a block of iron with circular cylindrical slots. The block of iron has a width of 14 in., a height of 16 in., and a length of 20 in. The number of cylinders drilled out of the block is determined by the weight of the leftover block, which must be less than 1,000 lb.
- a. If iron has a weight of roughly 491 lb/ft^3 , how many cylinders with the same height as the block and with radius 2 in. must be drilled out of the block in order for the remaining solid to weigh less than 1,000 lb.?

$$\begin{aligned} \text{WEIGHT OF BLOCK} &= \left(\frac{14 \times 16 \times 20 \text{ in}^3}{12^3 \frac{\text{in}^3}{\text{ft}^3}} \right) \cdot \left(491 \frac{\text{lb}}{\text{ft}^3} \right) \approx 1273 \text{ lbs} \\ \text{WEIGHT OF EACH DRILLED OUT CYLINDER} &= \left(\frac{64\pi \text{ in}^3}{12^3 \frac{\text{in}^3}{\text{ft}^3}} \right) \left(491 \frac{\text{lb}}{\text{ft}^3} \right) \approx 57 \text{ lbs} \\ \text{NUMBER OF CYLINDERS NEEDED TO BE DRILLED OUT} &= \frac{(1273 - 1000)}{57} \approx 4.8 \\ &\text{5 CYLINDERS MUST BE DRILLED OUT TO REDUCE THE WEIGHT TO NO MORE THAN 1000 lbs.} \end{aligned}$$

- b. If iron ore costs \$115 per ton (1 ton = 2200 lb.) and the price of each part is based solely on its weight of iron, how many parts can be purchased with \$1,500? Explain your answer.

$$\text{REMAINING SOLID} = [1273 - 5(57)] \text{ lbs} = 988 \text{ lbs.}$$

$$\left(\frac{\$115}{1 \text{ ton}} \right) \left(\frac{1 \text{ ton}}{2200 \text{ lbs}} \right) \left(\frac{988 \text{ lbs}}{1 \text{ block}} \right) \approx 29 \text{ blocks}$$

Approximately 29 blocks
can be purchased with
\$1500.

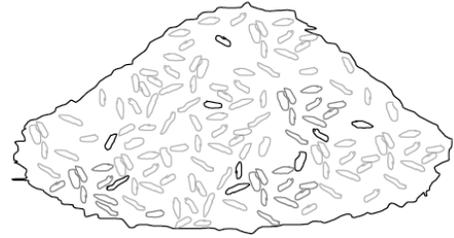
4. Rice falling from an open bag piles up into a figure conical in shape with an approximate radius of 5 centimeters.

- a. If the angle formed by the slant of the pile with the base is roughly 30° , write an expression that represents the volume of rice in the pile.

$$\tan 30 = \frac{h}{5}$$

$$5 \tan 30 = h$$

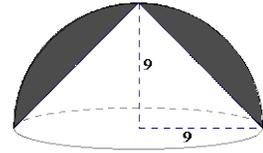
$$\begin{aligned} \text{VOLUME} &= \frac{\pi 5^2 (5 + \tan 30)}{3} \\ &= \frac{125 \pi \tan 30}{3} \end{aligned}$$



- b. If there are approximately 20 grains of rice in a cubic centimeter, approximately how many grains of rice are in the pile? Round to the nearest whole grain.

$$\begin{aligned} &\left(\frac{125 \pi \tan 30 \text{ cm}^3}{3} \right) \left(\frac{20 \text{ grains}}{1 \text{ cm}^3} \right) \\ &= \left(\frac{2500 \pi \tan 30}{3} \right) \text{ grains of rice} \\ &\approx 1511 \text{ grains of rice} \end{aligned}$$

5. In a solid hemisphere, a cone is removed as shown. Calculate the volume of the resulting solid. In addition to your solution, provide an explanation of the strategy you used in your solution.



Let V_1 be the volume of the hemisphere.

$$V_1 = \frac{1}{2} \left(\frac{4}{3} \right) \pi \cdot 9^3$$

$$= 486\pi$$

Let V_2 be the volume of the cone.

$$V_2 = \frac{1}{3} \pi \cdot 9^3$$

$$= 243\pi$$

Then the volume of the resulting solid is found by

$$486\pi - 243\pi = 243\pi$$

6. Describe the shape of the cross-section of each of the following objects.

Right circular cone:

- a. Cut by a plane through the vertex and perpendicular to the base

An isosceles triangle

Square pyramid:

- b. Cut by a plane through the vertex and perpendicular to the base

An isosceles triangle

- c. Cut by a vertical plane that is parallel to an edge of the base but not passing through the vertex

A trapezoid

Sphere with radius r :

- d. Describe the radius of the circular cross-section created by a plane through the center of the sphere

The radius of the cross-section will be equal in length to the radius of the sphere.

- e. Describe the radius of the circular cross-section cut by a plane that does not pass through the center of the sphere

The radius of the sphere is longer than the radius of the circular cross-section.

Triangular Prism:

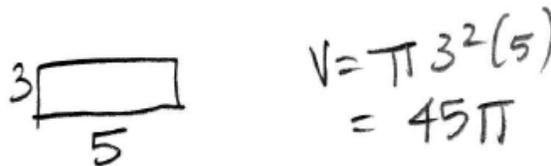
- f. Cut by a plane parallel to a base

A triangle

- g. Cut by a plane parallel to a face

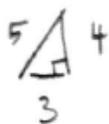
A rectangle

7. a. A 3×5 rectangle is revolved about one of its sides of length 5 to create a solid of revolution. Find the volume of the solid.



$$V = \pi 3^2 (5) = 45\pi$$

- b. A 3-4-5 right triangle is revolved about a leg of length 4 to create a solid of revolution. Describe the solid.



THE SOLID WILL BE A RIGHT CIRCULAR CONE WITH RADIUS 3 AND HEIGHT 4.

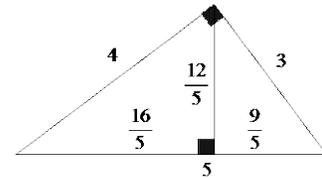
- c. A 3-4-5 right triangle is revolved about its legs to create two solids. Find each of the volumes of the two solids created.



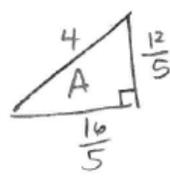
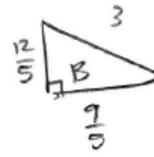
$$V = \frac{1}{3} \pi 3^2(4) = 12\pi$$


$$V = \frac{1}{3} \pi 4^2(3) = 16\pi$$

- d. Show that the volume of the solid created by revolving a 3-4-5 triangle about its hypotenuse is $\frac{48}{5}\pi$.



$$V_A = \frac{1}{3} \pi \left(\frac{16}{5}\right) \left(\frac{12}{5}\right)^2$$

$$V_B = \frac{1}{3} \pi \left(\frac{12}{5}\right)^2 \left(\frac{9}{5}\right)$$



$$V_A + V_B = \frac{1}{3} \pi \frac{16}{5} \left(\frac{12}{5}\right)^2 + \frac{1}{3} \pi \left(\frac{12}{5}\right)^2 \left(\frac{9}{5}\right)$$

$$= \left[\frac{1}{3} \pi \left(\frac{12}{5}\right)^2 \right] \left(\frac{16}{5} + \frac{9}{5} \right)$$

$$= \frac{1}{3} \pi \left(\frac{12}{5}\right)^2 (5)$$

$$= \frac{48}{5} \pi$$