

Name \_\_\_\_\_

Date \_\_\_\_\_

1. Solve the following equations for  $x$ . Write your answer in set notation.

a.  $3x - 5 = 16$

b.  $3(x + 3) - 5 = 16$

c.  $3(2x - 3) - 5 = 16$

d.  $6(x + 3) - 10 = 32$

e. Which two equations above have the same solution set? Write a sentence explaining how the properties of equality can be used to determine the pair without having to find the solution set for each.

2. Let  $c$  and  $d$  be real numbers.
- a. If  $c = 42 + d$  is true, then which is greater:  $c$  or  $d$ , or are you not able to tell? Explain how you know your choice is correct.
- b. If  $c = 42 - d$  is true, then which is greater:  $c$  or  $d$ , or are you not able to tell? Explain how you know your choice is correct.

3. If  $a < 0$  and  $c > b$ , circle the expression that is greater:

$$a(b - c) \quad \text{or} \quad a(c - b)$$

Use the properties of inequalities to explain your choice.

4. Solve for  $x$  in each of the equations or inequalities below, and name the property and/or properties used:

a.  $\frac{3}{4}x = 9$

b.  $10 + 3x = 5x$

c.  $a + x = b$

d.  $cx = d$

e.  $\frac{1}{2}x - g < m$

f.  $q + 5x = 7x - r$

g.  $\frac{3}{4}(x + 2) = 6(x + 12)$

h.  $3(5 - 5x) > 5x$

5. The equation  $3x + 4 = 5x - 4$  has the solution set  $\{4\}$ .

a. Explain why the equation  $(3x + 4) + 4 = (5x - 4) + 4$  also has the solution set  $\{4\}$ .

- b. In part (a), the expression  $(3x + 4) + 4$  is equivalent to the expression  $3x + 8$ . What is the definition of equivalent expressions? Why does changing an expression on one side of an equation to an equivalent expression leave the solution set unchanged?

- c. When we square both sides of the original equation, we get the following new equation:

$$(3x + 4)^2 = (5x - 4)^2.$$

Show that 4 is still a solution to the new equation. Show that 0 is also a solution to the new equation but is not a solution to the original equation. Write a sentence that describes how the solution set to an equation may change when both sides of the equation are squared.

- d. When we replace  $x$  by  $x^2$  in the original equation, we get the following new equation:

$$3x^2 + 4 = 5x^2 - 4.$$

Use the fact that the solution set to the original equation is  $\{4\}$  to find the solution set to this new equation.

6. The Zonda Information and Telephone Company (ZI&T) calculates a customer's total monthly cell phone charge using the formula,

$$C = (b + rm)(1 + t),$$

where  $C$  is the total cell phone charge,  $b$  is a basic monthly fee,  $r$  is the rate per minute,  $m$  is the number of minutes used that month, and  $t$  is the tax rate.

Solve for  $m$ , the number of minutes the customer used that month.



8. Alexis is modeling the growth of bacteria for an experiment in science. She assumes that there are  $B$  bacteria in a Petri dish at 12:00 noon. In reality, each bacterium in the Petri dish subdivides into two new bacteria *approximately* every 20 minutes. However, for the purposes of the model, Alexis assumes that each bacterium subdivides into two new bacteria *exactly* every 20 minutes.
- a. Create a table that shows the total number of bacteria in the Petri dish at  $\frac{1}{3}$  hour intervals for 2 hours starting with time 0 to represent 12:00 noon.
- b. Write an equation that describes the relationship between total number of bacteria  $T$  and time  $h$  in hours, assuming there are  $B$  bacteria in the Petri dish at  $h = 0$ .
- c. If Alexis starts with 100 bacteria in the Petri dish, draw a graph that displays the total number of bacteria with respect to time from 12:00 noon ( $h = 0$ ) to 4:00 p.m. ( $h = 4$ ). Label points on your graph at time  $h = 0, 1, 2, 3, 4$ .

- d. For her experiment, Alexis plans to add an anti-bacterial chemical to the Petri dish at 4:00 p.m. that is supposed to kill 99.9% of the bacteria instantaneously. If she started with 100 bacteria at 12:00 noon, how many live bacteria might Alexis expect to find in the Petri dish right after she adds the anti-bacterial chemical?
9. Jack is 27 years older than Susan. In 5 years, he will be 4 times as old as she is.
- a. Find the present ages of Jack and Susan.
- b. What calculations would you do to check if your answer is correct?

10.

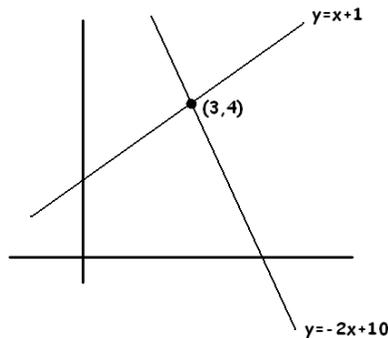
a. Find the product:  $(x^2 - x + 1)(2x^2 + 3x + 2)$ .

b. Use the results of part (a) to factor 21,112 as a product of a two-digit number and a three-digit number.

11. Consider the following system of equations with the solution  $x = 3, y = 4$ .

Equation A1:  $y = x + 1$

Equation A2:  $y = -2x + 10$



a. Write a unique system of two linear equations with the same solution set. This time make both linear equations have positive slope.

Equation B1: \_\_\_\_\_

Equation B2: \_\_\_\_\_

- b. The following system of equations was obtained from the original system by adding a multiple of equation A2 to equation A1.

Equation C1:  $y = x + 1$

Equation C2:  $3y = -3x + 21$

What multiple of A2 was added to A1?

- c. What is the solution to the system given in part (b)?

- d. For any real number  $m$ , the line  $y = m(x - 3) + 4$  passes through the point  $(3, 4)$ .

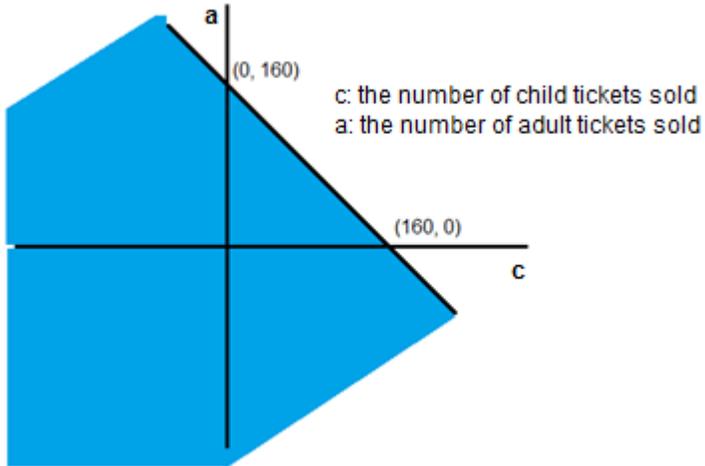
Is it certain, then, that the system of equations

Equation D1:  $y = x + 1$

Equation D2:  $y = m(x - 3) + 4$

has only the solution  $x = 3, y = 4$ ? Explain.

12. The local theater in Jamie's home town has a maximum capacity of 160 people. Jamie shared with Venus the following graph and said that the shaded region represented all the possible combinations of adult and child tickets that could be sold for one show.



- a. Venus objected and said there was more than one reason that Jamie's thinking was flawed. What reasons could Venus be thinking of?

- b. Use equations, inequalities, graphs, and/or words to describe for Jamie the set of all possible combinations of adult and child tickets that could be sold for one show.
- c. The theater charges \$9 for each adult ticket and \$6 for each child ticket. The theater sold 144 tickets for the first showing of the new release. The total money collected from ticket sales for that show was \$1,164. Write a system of equations that could be used to find the number of child tickets and the number of adult tickets sold, and solve the system algebraically. Summarize your findings using the context of the problem.

A Progression Toward Mastery

Assessment Task Item		STEP 1 Missing or incorrect answer and little evidence of reasoning or application of mathematics to solve the problem.	STEP 2 Missing or incorrect answer but evidence of some reasoning or application of mathematics to solve the problem.	STEP 3 A correct answer with some evidence of reasoning or application of mathematics to solve the problem, <u>OR</u> an incorrect answer with substantial evidence of solid reasoning or application of mathematics to solve the problem.	STEP 4 A correct answer supported by substantial evidence of solid reasoning or application of mathematics to solve the problem.
1	a–d <b>A-REI.A.1</b>	Student gave a short incorrect answer or left the question blank.	Student showed at least one correct step, but the solution was incorrect.	Student solved the equation correctly (every step that was shown was correct) but did not express the answer as a solution set.	Student solved the equation correctly (every step that was shown was correct) and expressed the answer as a solution set.
	e <b>A-SSE.A.1b</b> <b>A-REI.B.3</b>	Student did not answer or answered incorrectly with something other than (b) and (d).	Student answered (b) and (d) but did not demonstrate solid reasoning in the explanation.	Student answered (b) and (d) but made minor misstatements in the explanation.	Student answered (b) and (d) and articulated solid reasoning in the explanation.
2	a <b>A-CED.A.3</b>	Student responded incorrectly or left the question blank.	Student responded correctly that (c) must be greater but did not use solid reasoning to explain the answer.	Student responded correctly that (c) must be greater but gave an incomplete or slightly incorrect explanation of why.	Student responded correctly that (c) must be greater and supported the statement with solid, well-expressed reasoning.
	b <b>A-CED.A.3</b>	Student responded incorrectly or left the question blank.	Student responded correctly that there is no way to tell but did not use solid reasoning to explain the answer.	Student responded correctly that there is no way to tell but gave an incomplete or slightly incorrect explanation of why.	Student responded correctly that there is no way to tell and supported the statement with solid, well-expressed reasoning.

3	<b>A-SSE.A.1b</b>	Student responded incorrectly or left the question blank.	Student responded correctly by circling the expression on the left but did not use solid reasoning to explain the answer.	Student responded correctly by circling the expression on the left but gave limited explanation or did not use the properties of inequality in the explanation.	Student responded correctly by circling the expression on the left and gave a complete explanation that used the properties of inequality.
4	<b>a–h</b> <b>A-REI.A.1</b> <b>A-REI.B.3</b>	Student answered incorrectly with no correct steps shown.	Student answered incorrectly but had one or more correct steps.	Student answered correctly but did not correctly identify the property or properties used.	Student answered correctly and correctly identified the property or properties used.
5	<b>a</b> <b>A-REI.A.1</b>	Student did not answer or demonstrated incorrect reasoning throughout.	Student demonstrated only limited reasoning.	Student demonstrated solid reasoning but fell short of a complete answer or made a minor misstatement in the answer.	Student answer was complete and demonstrated solid reasoning throughout.
	<b>b</b> <b>A-REI.A.1</b>	Student did not answer or did not demonstrate understanding of what the question was asking.	Student made more than one misstatement in the definition.	Student provided a mostly correct definition with a minor misstatement.	Student answered completely and used a correct definition without error or misstatement.
	<b>c</b> <b>A-REI.A.1</b>	Student made mistakes in both verifications and demonstrated incorrect reasoning or left the question blank.	Student conducted both verifications but fell short of articulating reasoning to answer the question.	Student conducted both verifications and articulated valid reasoning to answer the question but made a minor error in the verification or a minor misstatement in the explanation.	Student conducted both verifications without error and articulated valid reasoning to answer the question.
	<b>d</b> <b>A-REI.A.1</b>	Student answered incorrectly or did not answer.	Student identified one or both solutions but was unable to convey how the solutions could be found using the fact that 4 is a solution to the original equation.	Student identified only one solution correctly but articulated the reasoning of using the solution to the original equation to find the solution to the new equation.	Student identified both solutions correctly and articulated the reasoning of using the solution to the original equation to find the solution to the new equation.

6	<b>A-CED.A.4</b>	Student did not answer or showed no evidence of reasoning.	Student made more than one error in the solution process but showed some evidence of reasoning.	Student answer showed valid steps but with one minor error.	Student answered correctly.
7	<b>a–c</b> <b>A-CED.A.3</b>	Student was unable to answer any portion correctly.	Student answered one part correctly or showed some evidence of reasoning in more than one part.	Student showed solid evidence of reasoning in every part but may have made minor errors.	Student answered every part correctly and demonstrated and expressed valid reasoning throughout.
8	<b>a</b> <b>A-CED.A.2</b>	Student provided no table or a table with multiple incorrect entries.	Student provided a data table that was incomplete or had more than one minor error.	Student provided a data table that was complete but may have had one error or slightly inaccurate headings.	Student provided a data table that was complete and correct with correct headings.
	<b>b</b> <b>A-CED.A.2</b>	Student provided no equation or an equation that did not represent exponential growth.	Student provided an incorrect equation but one that modeled exponential growth.	Student provided a correct answer in the form of $T = B(2)^{3h}$ .	Student provided a correct answer in the form of $T = B8^h$ or in more than one form, such as $T = B(2)^{3h}$ and $T = B8^h$ .
	<b>c</b> <b>A-CED.A.2</b>	Student provided no graph or a grossly inaccurate graph.	Student provided a graph with an inaccurate shape but provided some evidence of reasoning in labeling the axes and/or data points.	Student created a graph with correct general shape but may have left off or made an error on one or two axes or data points.	Student created a complete graph with correctly labeled axes and correctly labeled data points (or a data table) showing the values for $h = 0, 1, 2, 3, 4$ .
	<b>d</b> <b>A-CED.A.2</b>	Student provided no answer or an incorrect answer with no evidence of reasoning in arriving at the answer.	Student provided limited evidence of reasoning and an incorrect answer.	Student answered that 409.6 bacteria would be alive.	Student answered that 410, or about 410, bacteria would be alive.
9	<b>a</b> <b>A-CED.A.1</b>	Student wrote incorrect equations or did not provide equations.	Student answers were incorrect, but at least one of the equations was correct. Student may have made a gross error in the solution, made more	Both equations were correct, but student made a minor mistake in finding the solution.	Both equations were correct and student solved them correctly to arrive at the answer that Jack is 31 and Susan is 4.

			than one minor error in the solution process, or may have had one of the two equations incorrect.		
	<b>b</b> <b>A-REI.B.3</b>	Student did not answer or gave a completely incorrect answer.	Student articulated only one of the calculations correctly.	Student articulated the two calculations but with a minor misstatement in one of the descriptions.	Student articulated both calculations correctly.
<b>10</b>	<b>a–b</b> <b>A-APR.A.1</b>	Student work was blank or demonstrated no understanding of multiplication of polynomials, nor how to apply part (a) to arrive at an answer for part (b).	Student made more than one error in the multiplication but demonstrated some understanding of multiplication of polynomials. Student may not have been able to garner or apply information from part (a) to use in answering part (b) correctly.	Student demonstrated the ability to multiply the polynomials (expressing the product as a sum of monomials with like terms combined) and to apply the structure from part (a) to solve part (b). There may have been minor errors.	Student demonstrated the ability to multiply the polynomials (expressing the product as a sum of monomials with like terms combined) and to apply the structure from part (a) to solve part (b) as $91(232)$ .
<b>11</b>	<b>a</b> <b>A-REI.C.6</b>	Student was unable to demonstrate the understanding that two equations with $(3, 4)$ as a solution are needed.	Student provided two equations that have $(3, 4)$ as a solution (or attempted to provide such equations) but made one or more errors. Student may have provided an equation with a negative slope.	Student showed one minor error in the answer but attempted to provide two equations both containing $(3, 4)$ as a solution and both with positive slope.	Student provided two equations both containing $(3, 4)$ as a solution and both with positive slope.
	<b>b</b> <b>A-REI.C.6</b>	Student was unable to identify the multiple correctly.	Student identified the multiple as 3.	N/A	Student correctly identified the multiple as 2.
	<b>c</b> <b>A-REI.C.6</b>	Student was unable to demonstrate even a partial understanding of how to find the solution to the system.	Student showed some reasoning required to find the solution but made multiple errors.	Student made a minor error in finding the solution point.	Student successfully identified the solution point as $(3, 4)$ .

	<p><b>d</b></p> <p><b>A-REI.C.5</b> <b>A-REI.C.6</b> <b>A-REI.D.10</b></p>	<p>Student was unable to answer or to support the answer with any solid reasoning.</p>	<p>Student concluded yes or no but was only able to express limited reasoning in support of the answer.</p>	<p>Student correctly explained that all the systems would have the solution point (3, 4) but incorrectly assumed this is true for all cases of <math>m</math>.</p>	<p>Student correctly explained that while in most cases this is true, if <math>m = 1</math>, the two lines are coinciding lines, resulting in a solution set consisting of all the points on the line.</p>
12	<p><b>a</b></p> <p><b>MP.2</b> <b>A-REI.D.12</b></p>	<p>Student was unable to articulate any sound reasons.</p>	<p>Student was only able to articulate one sound reason.</p>	<p>Student provided two sound reasons but made minor errors in the expression of reasoning.</p>	<p>Student was able to articulate at least two valid reasons. Valid reasons include the following: the graph assumes <math>x</math> could be less than zero, the graph assumes <math>y</math> could be less than zero, the graph assumes <math>a</math> and <math>b</math> could be non-whole numbers, the graph assumes 160 children could attend with no adults.</p>
	<p><b>b</b></p> <p><b>A-CED.A.2</b> <b>A-REI.D.10</b> <b>A-REI.D.12</b></p>	<p>Student was unable to communicate a relevant requirement of the solution set.</p>	<p>Student provided a verbal description that lacked precision and accuracy but demonstrated some reasoning about the solution within the context of the problem.</p>	<p>Student made minor errors in communicating the idea that both (a) and (b) must be whole numbers whose sum is less than or equal to 160.</p>	<p>Student communicated effectively that both (a) and (b) must be whole numbers whose sum is less than or equal to 160.</p>
	<p><b>c</b></p> <p><b>A-CED.A.2</b> <b>A-REI.C.6</b></p>	<p>Student was unable to demonstrate any substantive understanding in how to create the equations and solve the system of equations.</p>	<p>Student made multiple errors in the equations and/or solving process but demonstrated some understanding of how to create equations to represent a context and/or solve the system of equations.</p>	<p>Student made minor errors in the equations but solved the system accurately, or the student created the correct equations but made a minor error in solving the system of equations.</p>	<p>Student correctly wrote the equations to represent the system. Student solved the system accurately and summarized by defining or describing the values of the variable in the context of the problem (i.e., that there were 100 adult tickets and 44 child tickets sold).</p>

Name \_\_\_\_\_

Date \_\_\_\_\_

1. Solve the following equations for  $x$ . Write your answer in set notation.

a.  $3x - 5 = 16$

$$\begin{aligned} 3x &= 21 && \text{Solution set: } \{7\} \\ x &= 7 \end{aligned}$$

b.  $3(x + 3) - 5 = 16$

$$\begin{aligned} 3x + 9 - 5 &= 16 && \text{Solution set: } \{4\} \\ 3x &= 12 \\ x &= 4 \end{aligned}$$

c.  $3(2x - 3) - 5 = 16$

$$\begin{aligned} 6x - 9 - 5 &= 16 && \text{Solution set: } \{5\} \\ 6x - 14 &= 16 \\ 6x &= 30 \\ x &= 5 \end{aligned}$$

d.  $6(x + 3) - 10 = 32$

$$\begin{aligned} 6x + 18 - 10 &= 32 && \text{Solution set: } \{4\} \\ 6x &= 24 \\ x &= 4 \end{aligned}$$

2. Which two equations above have the same solution set? Write a sentence explaining how the properties of equality can be used to determine the pair without having to find the solution set for each.

*Problems (b) and (d) have the same solution set. The expressions on each side of the equal sign for (d) are twice those for (b). So, if (left side) = (right side) is true for only some  $x$ -values, then  $2(\text{left side}) = 2(\text{right side})$  will be true for exactly the same  $x$ -values. Or simply, applying the multiplicative property of equality does not change the solution set.*

2. Let  $c$  and  $d$  be real numbers.

- a. If  $c = 42 + d$  is true, then which is greater:  $c$  or  $d$  or are you not able to tell? Explain how you know your choice is correct.

*$c$  must be greater because  $c$  is always 42 more than  $d$*

- b. If  $c = 42 - d$  is true, then which is greater:  $c$  or  $d$  or are you not able to tell? Explain how you know your choice is correct.

*There is no way to tell. We only know that the sum of  $c$  and  $d$  is 42. If  $d$  were 10,  $c$  would be 32 and, therefore, greater than  $d$ . But if  $d$  were 40,  $c$  would be 2 and, therefore, less than  $d$ .*

3. If  $a < 0$  and  $c > b$ , circle the expression that is greater:

$a(b - c)$  or  $a(c - b)$

Use the properties of inequalities to explain your choice.

*Since  $c > b$ ,  
It follows that  $0 > b - c$   
And since  $a < 0$ ,  $a$  is negative  
And the product of two negatives will be  
a positive.*

*Since  $c > b$ ,  
it follows that  $c - b > 0$ .  
so  $(c - b)$  is positive. And since  $a$  is  
negative, the product of  
 $a \cdot (c - b) < a \cdot (b - c)$ .*

4. Solve for  $x$  in each of the equations or inequalities below and name the property and/or properties used:

a.  $\frac{3}{4}x = 9$

$$x = 9 \cdot \left(\frac{4}{3}\right)$$

$$x = 12$$

*Multiplication Property of Equality*

b.  $10 + 3x = 5x$

$$10 = 2x$$

$$5 = x$$

*Addition Property of Equality*  
*Multiplication Property of Equality*

c.  $a + x = b$

$$x = b - a$$

*Addition Property of Equality*

d.  $cx = d$

$$x = \frac{d}{c}, c \neq 0$$

*Multiplication Property of Equality*

e.  $\frac{1}{2}x - g < m$

$$\frac{1}{2}x < m + g$$

$$x < 2 \cdot (m + g)$$

*Addition Property of Equality*  
*Multiplication Property of Equality*

f.  $q + 5x = 7x - r$

$$q + r = 2x$$

$$\frac{(q+r)}{2} = x$$

*Addition Property of Equality*  
*Multiplication Property of Equality*

g.  $\frac{3}{4}(x + 2) = 6(x + 12)$

$$3 \cdot (x + 2) = 24 \cdot (x + 12)$$

Multiplication Property of Equality

$$3x + 6 = 24x + 288$$

Distributive Property

$$-\frac{282}{94} = x$$

Addition Property of Equality and Multiplication

$$-\frac{7}{94} = x$$

Property of Equality

$$-\frac{94}{7} = x$$

h.  $3(5 - 5x) > 5x$

$$15 - 15x > 5x$$

Distributive Property

$$15 > 20x$$

Addition Property of Inequality

$$\frac{3}{4} > x$$

Multiplication Property of Equality

5. The equation,  $3x + 4 = 5x - 4$ , has the solution set  $\{4\}$ .

a. Explain why the equation,  $(3x + 4) + 4 = (5x - 4) + 4$ , also has the solution set  $\{4\}$ .

*Since the new equation can be created by applying the additive property of equality, the solution set does not change.*

OR

*Each side of this equation is 4 more than the sides of the original equation. Whatever value(s) make  $3x + 4 = 5x - 4$  true would also make 4 more than  $3x + 4$  equal to 4 more than  $5x - 4$ .*

b. In part (a), the expression  $(3x + 4) + 4$  is equivalent to the expression  $3x + 8$ . What is the definition of equivalent algebraic expressions? Describe why changing an expression on one side of an equation to an equivalent expression leaves the solution set unchanged?

*Algebraic expressions are equivalent if (possibly repeated) use of the distributive, associative, and commutative properties, and/or the properties of rational exponents can be applied to one expression to convert it to the other expression.*

*When two expressions are equivalent, assigning the same value to  $x$  in both expressions will give an equivalent numerical expression, which then evaluates to the same number. Therefore, changing the expression to something equivalent will not change the truth value of the equation once values are assigned to  $x$ .*

- c. When we square both sides of the original equation, we get the following new equation:

$$(3x + 4)^2 = (5x - 4)^2.$$

Show that 4 is still a solution to the new equation. Show that 0 is also a solution to the new equation but is not a solution to the original equation. Write a sentence that describes how the solution set to an equation may change when both sides of the equation are squared.

$(3 \cdot 4 + 4)^2 = (5 \cdot 4 - 4)^2$  gives  $16^2 = 16^2$ , which is true.

$(3 \cdot 0 + 4)^2 = (5 \cdot 0 - 4)^2$  gives  $4^2 = (-4)^2$ , which is true.

But,  $(3 \cdot 0 + 4) = (5 \cdot 0 - 4)$  gives  $4 = -4$ , which is false.

When both sides are squared, you might introduce new numbers to the solution set because statements like  $4 = -4$  are false, but statements like  $4^2 = (-4)^2$  are true.

- d. When we replace  $x$  by  $x^2$  in the original equation, we get the following new equation:

$$3x^2 + 4 = 5x^2 - 4.$$

Use the fact that the solution set to the original equation is  $\{4\}$  to find the solution set to this new equation.

Since the original equation  $3x + 4 = 5x - 4$  was true when  $x = 4$ , the new equation  $3x^2 + 4 = 5x^2 - 4$  should be true when  $x^2 = 4$ . And,  $x^2 = 4$  when  $x = 2$ , so the solution set to the new equation is  $\{-2, 2\}$ .

6. The Zonda Information and Telephone Company (ZI&T) calculates a customer's total monthly cell phone charge using the formula,

$$C = (b + rm)(1 + t),$$

where  $C$  is the total cell phone charge,  $b$  is a basic monthly fee,  $r$  is the rate per minute,  $m$  is the number of minutes used that month, and  $t$  is the tax rate.

Solve for  $m$ , the number of minutes the customer used that month.

$$\begin{aligned} C &= b + bt + rm + rmt \\ C - b - bt &= m \cdot (r + rt) \\ \frac{C - b - bt}{r + rt} &= m \quad t \neq -1 \\ &\quad r \neq 0 \end{aligned}$$

7. Students and adults purchased tickets for a recent basketball playoff game. All tickets were sold at the ticket booth—season passes, discounts, etc. were not allowed.

Student tickets cost \$5 each, and adult tickets cost \$10 each. A total of \$4,500 was collected. 700 tickets were sold.

- a. Write a system of equations that can be used to find the number of student tickets,  $s$ , and the number of adult tickets,  $a$ , that were sold at the playoff game.

$$\begin{cases} 5s + 10a = 4500 \\ s + a = 700 \end{cases}$$

- b. Assuming that the number of students and adults attending would not change, how much more money could have been collected at the playoff game if the ticket booth charged students and adults the same price of \$10 per ticket?

$$\begin{aligned} 700 \times \$10 &= \$7000 \\ \$7000 - \$4500 &= \$2500 \text{ more} \end{aligned}$$

- c. Assuming that the number of students and adults attending would not change, how much more money could have been collected at the playoff game if the student price was kept at \$5 per ticket and adults were charged \$15 per ticket instead of \$10?

$$\begin{aligned} \text{First solve for } a \text{ \& } s \quad 5s + 10a &= 4500 \\ -5s - 5a &= -3500 \\ 5a &= 1000 \\ a &= 200 \\ s &= 500 \end{aligned}$$

$$\begin{aligned} \$5 \cdot (500) + \$15 \cdot (200) &= \$5500 \\ \$1000 \text{ more} \end{aligned}$$

OR

$$\$5 \text{ more per adult ticket } (200 \cdot \$5 = \$1000 \text{ more})$$

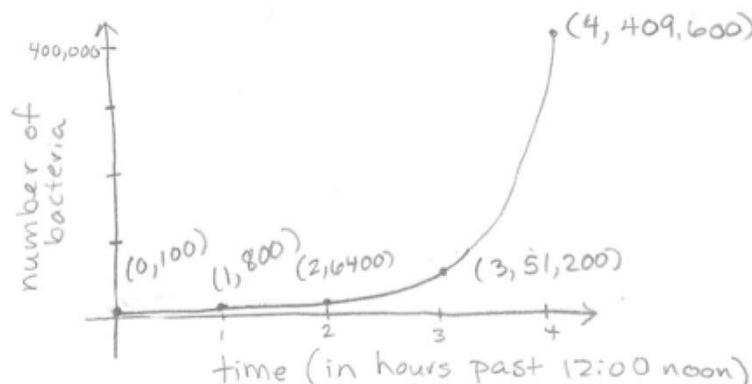
8. Alexis is modeling the growth of bacteria for an experiment in science. She assumes that there are  $B$  bacteria in a Petri Dish at 12:00 noon. In reality, each bacterium in the Petri dish subdivides into two new bacteria *approximately* every 20 minutes. However, for the purposes of the model, Alexis assumes that each bacterium subdivides into two new bacteria *exactly* every 20 minutes.
- a. Create a table that shows the total number of bacteria in the Petri dish at  $\frac{1}{3}$  hour intervals for 2 hours starting with time 0 to represent 12:00 noon.

Time	Number of Bacteria
0	$B$
$\frac{1}{3}$ hour	$2B$
$\frac{2}{3}$ hour	$4B$
1 hour	$8B$
$1 \frac{1}{3}$ hour	$16B$
$1 \frac{2}{3}$ hour	$32B$
2 hour	$64B$

- b. Write an equation that describes the relationship between total number of bacteria  $T$  and time  $h$  in hours, assuming there are  $B$  bacteria in the Petri dish at  $h=0$ .

$$T = B \cdot (2)^{3h} \text{ or } T = B \cdot 8^h$$

- c. If Alexis starts with 100 bacteria in the Petri dish, draw a graph that displays the total number of bacteria with respect to time from 12:00 noon ( $h = 0$ ) to 4:00 p.m. ( $h = 4$ ). Label points on your graph at time  $h = 0, 1, 2, 3, 4$ .



- d. For her experiment, Alexis plans to add an anti-bacterial chemical to the Petri dish at 4:00 p.m. that is supposed to kill 99.9% of the bacteria instantaneously. If she started with 100 bacteria at 12:00 noon, how many live bacteria might Alexis expect to find in the Petri dish right after she adds the anti-bacterial chemical?

$$(1 - 0.999) \cdot 409,600 = 409.6$$

*about 410 live bacteria*

9. Jack is 27 years older than Susan. In 5 years time he will be 4 times as old as she is.
- a. Find the present ages of Jack and Susan.

$$\begin{cases} J = S + 27 \\ J + 5 = 4 \cdot (S + 5) \end{cases}$$

$$\begin{aligned} S + 27 + 5 &= 4S + 20 \\ S + 32 &= 4S + 20 \\ 12 &= 3S \\ S &= 4 \end{aligned}$$

$$\begin{aligned} J &= 4 + 27 \\ J &= 31 \end{aligned}$$

*Jack is 31 and Susan is 4.*

- b. What calculations would you do to check if your answer is correct?

*Is Jack's age – Susan's age = 27?*

*Add 5 years to Jack's and Susan's ages and see if that makes Jack 4 times as old as Susan.*

10.

a. Find the product:  $(x^2 - x + 1)(2x^2 + 3x + 2)$

$$\begin{array}{r} 2x^4 + 3x^3 + 2x^2 - 2x^3 - 3x^2 - 2x + 2x^2 + 3x + 2 \\ 2x^4 + x^3 + x^2 + x + 2 \end{array}$$

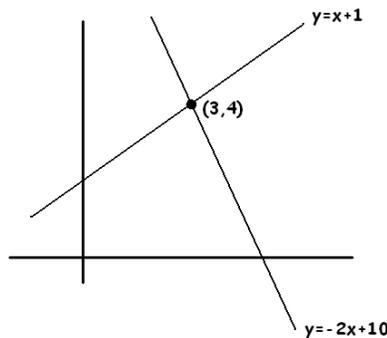
b. Use the results of part (a) to factor 21,112 as a product of a two-digit number and a three-digit number.

$$\begin{array}{r} (100 - 10 + 1) \cdot (200 + 30 + 2) \\ (91) \cdot (232) \end{array}$$

11. Consider the following system of equations with the solution  $x = 3, y = 4$ .

Equation A1:  $y = x + 1$

Equation A2:  $y = -2x + 10$



a. Write a unique system of two linear equations with the same solution set. This time make both linear equations have positive slope.

Equation B1:  $y = \frac{4}{5}x$

Equation B2:  $y = x + 1$

- b. The following system of equations was obtained from the original system by adding a multiple of equation A2 to equation A1.

Equation C1:  $y = x + 1$

Equation C2:  $3y = -3x + 21$

What multiple of A2 was added to A1?

*2 times A2 was added to A1.*

- c. What is the solution to the system given in part (b)?

*(3,4)*

- d. For any real number  $m$ , the line  $y = m(x - 3) + 4$  passes through the point  $(3,4)$ .

Is it certain, then, that the system of equations:

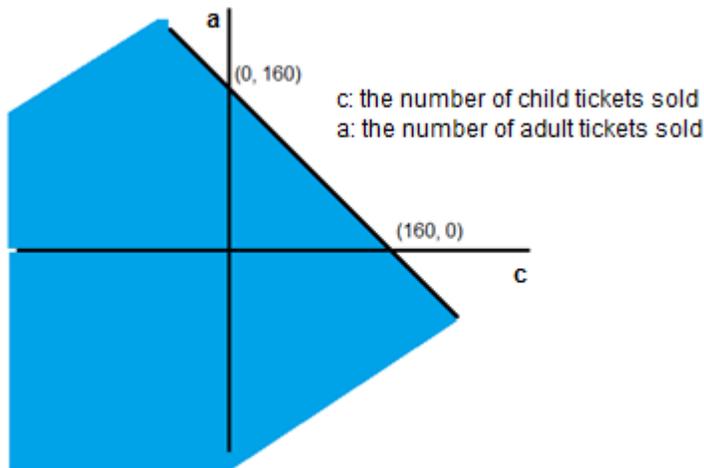
Equation D1:  $y = x + 1$

Equation D2:  $y = m(x - 3) + 4$

has only the solution  $x = 3, y = 4$ ? Explain.

*No. If  $m = 1$ , then the two lines have the same slope. Since both lines pass through the point  $(3,4)$ , and the lines are parallel; therefore, they coincide. There are infinite solutions. The solution set is all the points on the line. Any other non-zero value of  $m$  would create a system with the only solution of  $(3,4)$ .*

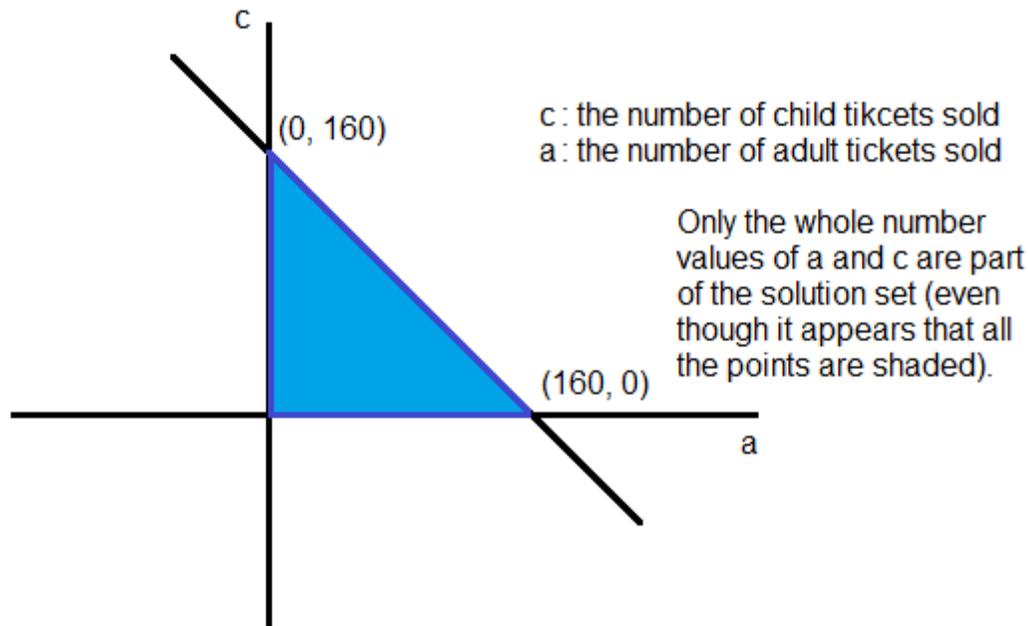
12. The local theater in Jamie's home town has a maximum capacity of 160 people. Jamie shared with Venus the following graph and said that the shaded region represented all the possible combinations of adult and child tickets that could be sold for one show.



- a. Venus objected and said there was more than one reason that Jamie's thinking was flawed. What reasons could Venus be thinking of?
1. The graph implies that the number of tickets sold could be a fractional amount, but really it only makes sense to sell whole number tickets.  $x$  and  $y$  must be whole numbers.
  2. The graph also shows that negative ticket amounts could be sold which does not make sense.

- b. Use equations, inequalities, graphs, and/or words to describe for Jamie the set of all possible combinations of adult and child tickets that could be sold for one show.

The system would be  $\begin{cases} a+c \leq 160 \\ a \geq 0 \\ c \geq 0 \end{cases}$  where  $a$  and  $c$  are whole numbers.



- c. The theater charges \$9 for each adult ticket and \$6 for each child ticket. The theater sold 144 tickets for the first showing of the new release. The total money collected from ticket sales for that show was \$1,164. Write a system of equations that could be used to find the number of child tickets and the number of adult tickets sold, and solve the system algebraically. Summarize your findings using the context of the problem.

$a$ : the number of adult tickets sold (must be a whole number)

$c$ : the number of child tickets sold (must be a whole number)

$$\begin{cases} 9a + 6c = 1164 \\ a + c = 144 \end{cases}$$

$$\begin{aligned} 9a + 6c &= 1164 \\ -6a - 6c &= -864 \\ 3a &= 300 \\ a &= 100, \quad c = 44 \end{aligned}$$

In all, 100 adult tickets and 44 child tickets were sold.